Exercise Sheet 4 Solutions

1. The following exercise is an example of a two-way ANOVA with a significant interaction where it does make sense to interpret the main effects directly.

A researcher wanted to investigate the effects of regular exercise and gender on heart rate. The factor Exercise has two levels, ‘Control’ and ‘Runners’ indicating whether an individual participates in regular exercise. The factor Gender has two levels ‘Male’ and ‘Female’. Heart rates (in beats per minute) were recorded during aerobic activity for 200 participants in each treatment group.

After the preliminary data analysis, a two-way ANOVA was carried out on the data set, followed by Tukey’s HSD post hoc test. The R output is shown below:

**Figure 1** **Figure 2**

 

**Figure 3**

Df Sum Sq Mean Sq F value Pr(>F)

Gender 1 45030 45030 185.980 < 2e-16 \*\*\*

Exercise 1 168432 168432 695.647 < 2e-16 \*\*\*

Gender:Exercise 1 1794 1794 7.409 0.00663 \*\*

Residuals 796 192730 242

**Figure 4**

Tukey multiple comparisons of means

95% family-wise confidence level

Fit: aov(formula = HR ~ Gender \* Exercise, data = running)

$Gender

diff lwr upr p adj

Male-Female -15.005 -17.16479 -12.84521 0

$Exercise

diff lwr upr p adj

Runners-Control -29.02 -31.17979 -26.86021 0

$`Gender:Exercise`

diff lwr upr p adj

Male:Control-Female:Control -18.000 -22.00595 -13.994054 0

Female:Runners-Female:Control -32.015 -36.02095 -28.009054 0

Male:Runners-Female:Control -44.025 -48.03095 -40.019054 0

Female:Runners-Male:Control -14.015 -18.02095 -10.009054 0

Male:Runners-Male:Control -26.025 -30.03095 -22.019054 0

Male:Runners-Female:Runners -12.010 -16.01595 -8.004054 0

**Figure 5**

omega\_sq(model1)

Gender Exercise Gender:Exercise

0.109712899 0.412000008 0.003801508

* 1. What are the null and alternative hypotheses associated with this analysis?

Let Factor A represent Gender then, αi represents the treatment effect associated with level *i* of Gender.

Let Factor B represent Exercise then, β j represents the treatment effect associated with level *j* of Exercise.

The hypotheses associated with this two-way ANOVA are:

H₀ : α₁ = α₂ = ... = αa = 0

HA : at least one αi ≠ 0

H₀ : β₁ = β₂ = .. .= βb=0

HA : at least one βj ≠ 0

H₀ : (αβ)ij = 0 for all i,j

HA : at least one (αβ)ij ≠ 0

* 1. Report the results of the two- way ANOVA including interpretation of the main and interaction effects.

The effect of regular exercise and gender on an individual’s heart rate during aerobic activity was investigated using a two-way ANOVA. The factor Exercise had two levels, *control* and *runners* and the factor Gender had two levels *male* and *female*. Heartrate was recorded for 200 individual from each treatment, giving an overall sample size of 800. The main effect of Exercise was significant, indicating that the mean heartrate of individuals who participated in regular exercise was significantly lower than the mean heartrate of those that did not participate in regular exercise F(1,796) = 695.647, MSE = 242, p < 0.001, ω2= 0.412. The main effect of Gender was also significant indicating that the mean heartrate for males was significantly lower than for females F(1,796) = 185.98, MSE = 242, p < 0.001, ω2= 0.1097. There was a significant interaction between the effects of Exercise and Gender on the measured heartrate F(1, 796) = 7.409, MSE = 242, p = .007, ω2=0.0038. The significant interaction between Gender and Exercise indicates that the effect of regular exercise on measured heartrate is different for different levels of Gender. The interaction plot shows that regular exercise had a greater impact on heartrate for females than males.

1. The data for this example comes from a farm-scale trial of animal diets. There are two factors: diet and supplement. Diet is a factor with three levels: barley, oats and wheat. Supplement is a factor with four levels: agrimore, control, supergain and supersupp. The response variable is weight gain after 6 weeks. The data from the experiment is available on Blackboard in the growth.txt file.

Carry out a two-way ANOVA on the data and write up a brief report including the following outputs and any conclusions that you draw.

* + Exploratory data analysis
  + ANOVA table
  + The effect size ω²
  + Model diagnostics

Out of the different diets, the greatest mean weight gain was for the barley diet, with a mean gain of 24.4 kg (see Table 1). The supplement with the largest mean gain (23.1 kg) was agrimore (see Table 2). The boxplots (see Fig. 1) and histograms (see Figs. 2 and 3) indicate that there are no potential outliers and there is no evidence of strong skew. Homogeneity of variance was assessed using the Fligner – Killeen test which found variance of gain did not differ significantly across different treatments, p-value = 0.4091 (11 d.f.).

An interaction plot showed that there was no interaction between diet and supplement (see Fig. 4), this was confirmed by the two-way ANOVA (Table 3). Diet had significant effect on weight gain F = (2,36) = 83.52, MSE = 1.72 p < 0.001, ω2= 0.636. The mean weight gain of animals fed a diet of barley was significantly greater than those fed wheat or oats. The mean weight gain of animals fed a diet of oats was significantly greater than those fed wheat (see Table 4). Supplement also had significant effect on weight gain F = (3,36) = 17.82, MSE = 1.72 p < 0.001, ω2= 0.19. The mean weight gain of animals given a supplement of agrimore or suppersupp was significantly greater than those given other supplements (see Table 5).

**Table 1**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Diet | Mean Gain | SD Gain | Max Gain | Min Gain | No. Obs. (n) |
| Barley | 24.4 | 2.27 | 29.0 | 20.9 | 16 |
| Oats | 21.3 | 1.67 | 24.9 | 19.0 | 16 |
| Wheat | 18.4 | 1.60 | 21.4 | 15.8 | 16 |

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Supplement | Mean Gain | SD Gain | Max Gain | Min Gain | No. Obs. (n) |
| agrimore | 23.1 | 3.18 | 29.0 | 18.0 | 12 |
| control | 20.4 | 2.71 | 24.7 | 16.1 | 12 |
| supergain | 19.7 | 2.54 | 23.8 | 15.8 | 12 |
| supersupp | 22.4 | 2.85 | 27.8 | 18.6 | 12 |

**Figure 1**



**Figure 2**



**Figure 3**



**Figure 4**



**Table 3**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | d.f. | Sum Sq | Mean sq | F- Value | p-value |
| Supplement | 3 | 287.17 | 30.63 | 17.82 | < 0.001 |
| Diet | 2 | 91.88 | 143.59 | 83.52 | < 0.001 |
| Diet: Supplement | 6 | 3.41 | 0.57 | 0.33 | 0.917 |
| Residuals | 36 | 61.89 | 1.72 |  |  |

**Table 4**

|  |  |  |  |
| --- | --- | --- | --- |
| Diet | Mean Gain | SD Gain | No. Obs. (n) |
| Barley | 24.4 | 2.27 | 16 |
| Oats | 21.3 | 1.67 | 16 |
| Wheat | 18.4 | 1.60 | 16 |

Note. All means were significantly different at the .05 level according to a Tukey’s HSD test.

**Table 5**

|  |  |  |  |
| --- | --- | --- | --- |
| Supplement | Mean Gain | SD Gain | No. Obs. (n) |
| agrimore | 23.1A | 3.18 | 12 |
| control | 20.4B | 2.71 | 12 |
| supergain | 19.7B | 2.54 | 12 |
| supersupp | 22.4A | 2.85 | 12 |

Note. Means sharing a letter in their superscript are not significantly different at the .05 level according to a Tukey’s HSD test.